TUTORIAL 2: Propositional and Predicate Logic

1. Define Tautology and Contradiction with examples. Prove that $P \rightarrow (P \vee Q)$ is Tautology without constructing truth table.
2. Verify the validity of the following statements:

   "All men are mortal. Socrates is a man. There for, Socrates is mortal".

3. State and prove the De Morgan's laws.

4. Use the laws of logic to show that $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg q$.

5. Construct the truth table for the following forms,
   (i) $(p \rightarrow (q \vee r)) \wedge ((q \rightarrow p) \wedge (p \vee r))$.
   (ii) $(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$
   (iii) $\neg(p \wedge q) \vee (p \leftrightarrow q)$
   (iv) $\neg[p \wedge (p \vee \neg q)]$
6. Test the validity of the logical consequences:
   All dog fetch.
   Ketty does not fetch.
   Therefore, Ketty is not a dog.

7. Use a truth table to determine whether the following statement form is valid:
   $x \rightarrow y$
   $x \rightarrow z$
   Therefore, $x \rightarrow y \vee z$.
8. Prove the following logical equivalence (use lows of logic).
   (i) $[(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)] \leftrightarrow p \wedge q$
   (ii) $(p \rightarrow q) \wedge [\neg p \wedge (r \vee \neg q)] \leftrightarrow \neg(q \vee p)$
9. If $M(x)$ is "x is man" $C(x)$ is "x is clever" then translate the following statements into English.
   (a) $\exists x(M(x) \rightarrow C(x))$ (b) $\forall x(M(x) \wedge C(x))$
10. Use truth table to obtain principal disjunctive normal form for the logical expression $(\neg p \vee \neg q) \wedge (\neg p \vee r)$.
11. Check the validity of the given argument "If I drive to work then I will arrive in time. I do not drive to work. Therefore, I will not arrive in time." using inference rules of ropositional logic
12. Give the converse and contra-positive of the conditional.
    If it rains, then I carry an umbrella.
13. Express the following proposition in to symbolic form
    (i) 'Either my program runs and it contains no bugs, or my program contains bugs'.
    (ii) 'Indians will win the world cup if their fielding improves'.
    (iii) 'If I am not in a good mood or I am not busy, then I will go for movie'.
    (iv) 'Unless he studies, he will fail in the examination'.
    (v) 'Raju is poor but happy'.
14. Let p be "Jack speaks French" and q be "Jack speaks English"
    Give a simple variable sentence which describes each of the following,
    (i) $p \vee q$
    (ii) $p \wedge q$
    (iii) $p \wedge \neg q$
    (iv) $\neg p \vee \neg q$
    (v) $\neg\neg p$
    (vi) $\neg(\neg p \wedge \neg q)$

15. If $p \to q$ is false, determine the truth value of $(\neg(p \wedge q)) \to q$.

16. If p and q are false, determine the truth value of $(p \vee q) \wedge (\neg p \vee \neg q)$.

17. Obtain DNF of the following form,
    (i) $(p \wedge (p \to q)) \to q$.
    (ii) $(p \to q) \wedge (\neg p \wedge q)$.

18. Obtain CNF of the following form,
    (i) $(\neg p \to r) \wedge (p \leftrightarrow q)$.
    (ii) $(p \wedge q) \vee (\neg p \wedge q \wedge r)$.

19. Compute the truth table of the statement, $(p \to q) \leftrightarrow (\neg q \to \neg p)$. Is this statement a tautology?

20. Write in symbols of the quantifier For all x, $x < 5$ or $x \geq 5$.

21. Consider the statement "All the invited guests were present for dinner" write negation of a quantified statement.
