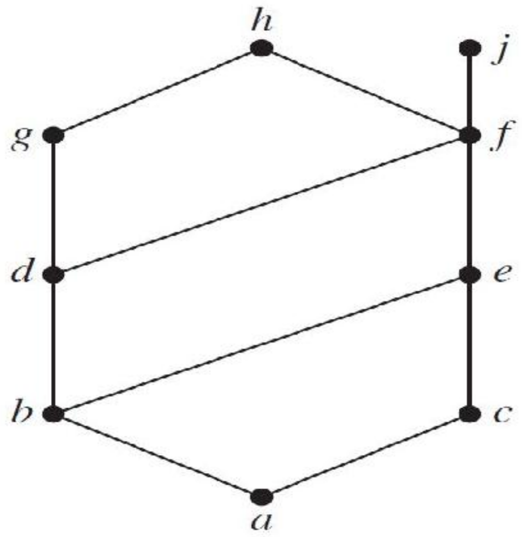
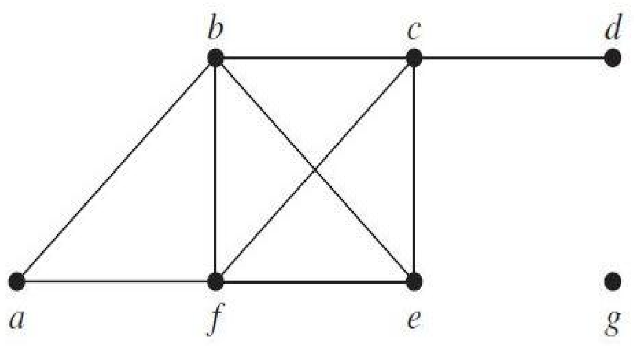
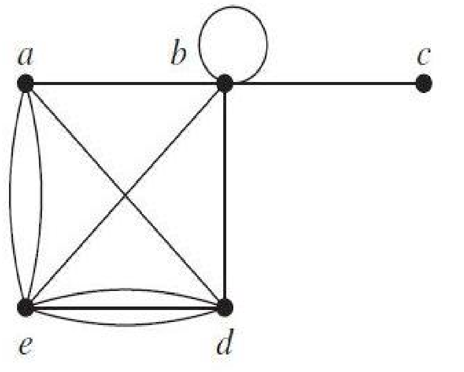
Discrete Mathematics. (3140708)

TUTORIAL 3: Relations and Partial ordering

<table>
<tr><td>1.</td><td>Define Relation. Let X= {1, 2, 3, 5} and R= {(x, y)/x>y}. Draw the graph of R and also give its matrix.</td></tr>
<tr><td>2.</td><td>Draw the Hasse diagrams of the following sets under the partial ordering relation “divides” and indicate those which are totally ordered.
(i) {1, 2, 3, 5}
(ii) {3, 5, 15}
(iii) {2, 4, 8, 16}
(iv) {2, 3, 6, 12, 24, 36}</td></tr>
<tr><td>3.</td><td>(i) Find the lub and glb of {b,d,g} if they exists, in the poset shown in Figure bellow.


(ii) Find the Fibonacci numbers $f_{2,}$ f_3, f_4, f_5 and f_6.</td></tr>
<tr><td>4.</td><td>What are the degrees and what are the neighbourhoods of the vertices in the graphs G and H displayed in Figure below?


G


H</td></tr>
<tr><td>5.</td><td>Consider the following relations on {1, 2, 3, 4}
$R_1 = \{(1,2),(2,2),(3,4)(4,1)\},\ R_2 = \{(1,2),(1,3),(4,2)\},\ R_3 = \{(3,4)\}$
$R_4 = \{(1,1)(1,2),(1,4),(2,1),(2,2),(3,3),(4,1),(4,4)\},\ R_5 = \{(1,1),(1,2),(2,1)\}$
$R_6 = \{(1,1)(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,3),(3,4),(4,4)\}$
Which of these relations are reflexive, transitive, symmetric and anti-symmetric?</td></tr>
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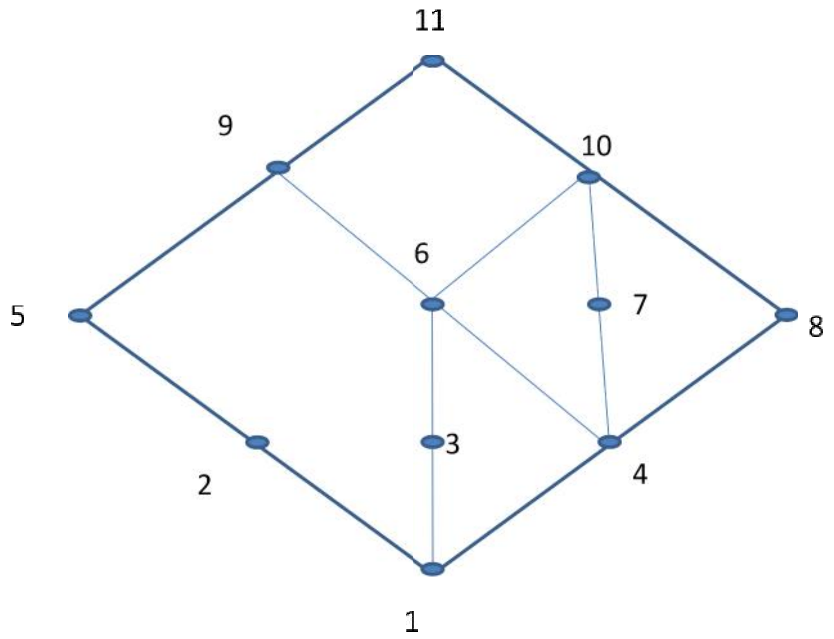
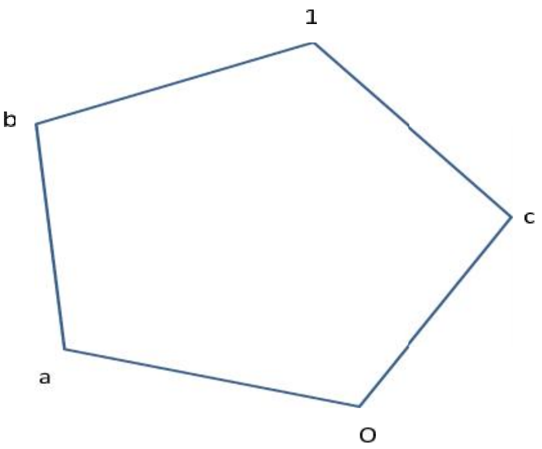
6.	Compute $A \vee B, A \wedge B, A^T B^T, AB$ for $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
7.	Find the general solution of $a_n + 5a_{(n-1)} + 6a_{(n-2)} = 3n^2$.
8.	Determine whether the following poset is Boolean algebras. Justify your answer. A= {1, 2, 3, 6} with divisibility.
9.	Let A= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}, be the poset whose Hasse diagram is shown in Figure below. Find LUB and GLB of B= {6, 7, 10} if they exist. 
10	Let A= {a, b, c} and let, $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Determine whether R is an equivalence relation
11.	Let X={1, 2, 3, 4,5} and R, S, T be the relations as follows: $R = \{(x, y) / x + y = 5\}$, $S = \{(1,2), (3,4), (2,2)\}$, $T = \{(4,2), (2,5), (3,1), (1,3)\}$ (i) Write Properties of R. (ii) Write matrix or R. (iii) Find $S \circ T$, $R \circ S$ and $S \circ R$.
12.	Define equivalence relation. Let Z be the set of integers and R be the relation called "Congruence modulo 5" defined by $R = \{(x, y) / (x - y)$ is divisible by $5\}$ show that R is an equivalence relation. Determine the equivalence classes generated by the elements of Z.
13.	Show that this is a complimented lattice 

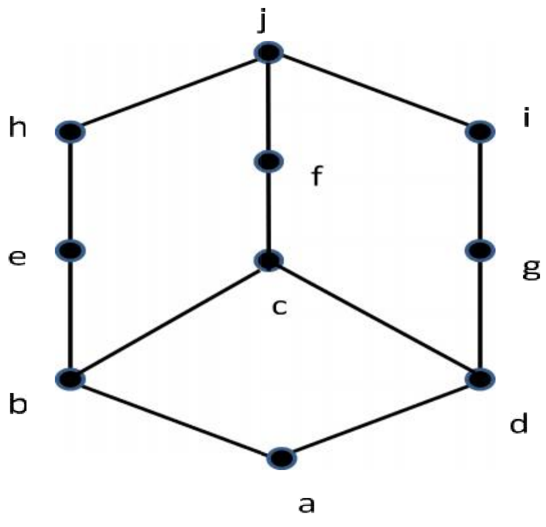
14. Consider the set of natural numbers N with the relation D - divides, and then show that an algebraic structure (N, D) is a POSET.

15. Given A={1, 2, 3, 4} and a relation R on A given by R={(4,3), (2,2), (2,1), (3,1), (1,2)}

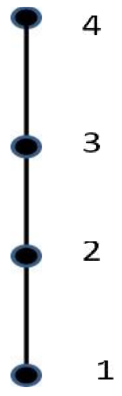
(i) Show that R is not transitive.

(ii) Find Transitive closer of R by Warshall's algorithm.

16. Determine whether the following Hasse diagram represent Boolean algebra.



(i)



(ii)

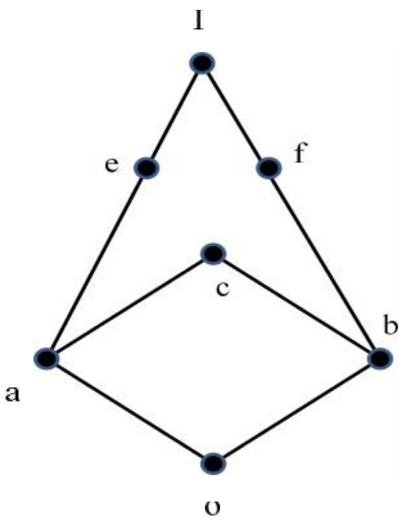
17. Let A={1,2,3} and let R and S be relations on A, whose matrices are given. Compute $M_{S \circ R}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

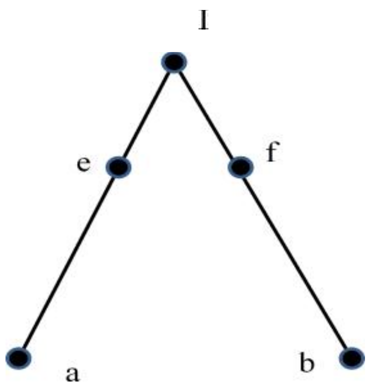
18. Determine Hasse diagram of the relation on A={1, 2, 3, 4, 5}, whose matrix is

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ M_R = \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

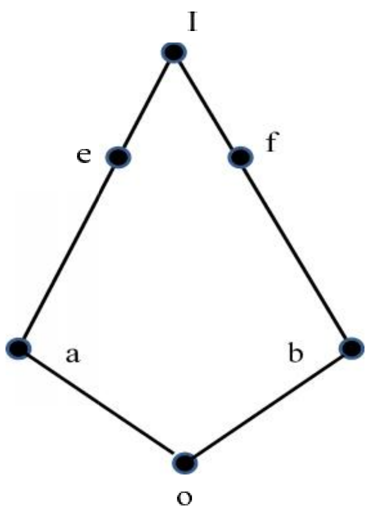
19. Consider the lattice shown in below Fig. **(a),** Determine whether or not each of the Fig. **(b)** and **(c)** is a sub-lattice of **(a)**.



(a)



(b)



(c)

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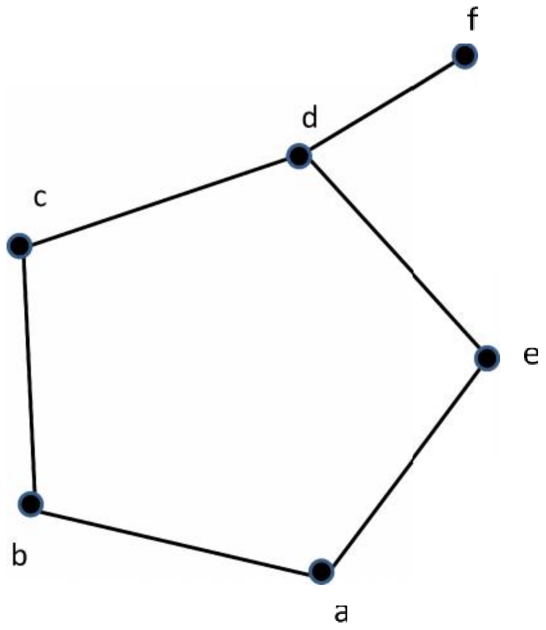
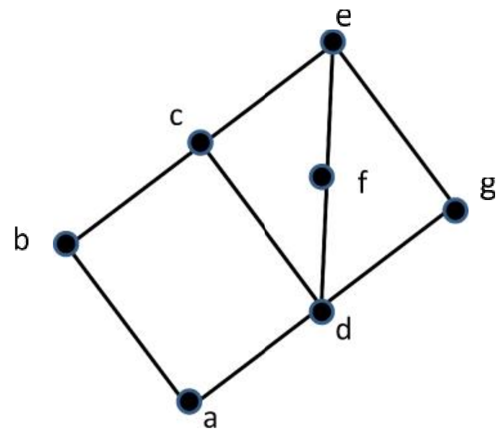
<tr><td>20.</td><td>Draw the Hasse diagram of D_{24} .i.e. set of integers which divide 24 with ‘divisibility’.</td></tr>
<tr><td>21.</td><td>Determine whether each lattice is distributive, complimented or both. Justify your answer.


(i)


(ii)</td></tr>
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