Discrete Mathematics-3140708
Tutorial 1

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1. What is the power set of the set $\{0, 1, 2\}$?
2. What is the power set of the empty set? What is the power set of the set $\{\phi\}$?
3. Show that the Cartesian product $B \times A$ is not equal to the Cartesian product $A \times B$, where $A = \{1, 2\}$ and $B = \{a, b, c\}$?
4. What is the Cartesian product $A \times B \times C$, where $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{0, 1, 2\}$?
5. Find the power set of each of these sets, where a and b are distinct elements.
   (a) $\{a\}$
   (b) $\{a, b\}$
   (c) $\{\phi, \{\phi\}\}$
6. What is the Cartesian product $A \times B$, where A is the set of courses offered by the mathematics department at a university and B is the set of mathematics professors at this university? Give an example of how this Cartesian product can be used.
7. Suppose that $A \times B = \phi$, where A and B are sets. What can you conclude?
8. How many different elements does $A \times B$ have if A has m elements and B has n elements?
9. Find A^3 if
   (a) $A = \{a\}$
   (b) $A = \{0, a\}$
10. State and prove following set identities
    (a) Identity laws
    (b) Domination laws
    (c) Idempotent laws
    (d) Complementation law
    (e) Commutative laws
    (f) Associative laws
    (g) Distributive laws
    (h) De Morgan's laws
    (i) Absorption laws

(j) Compliment laws

11. Let $A = \{0, 2, 4, 6, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{0, 3, 6, 9\}$. What are $A \cup B \cup C$ and $A \cap B \cap C$?

12. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and the ordering of elements of U has the elements in increasing order; that is, $a_i = i$. What bit strings represent the subset of all odd integers in U, the subset of all even integers in U, and the subset of integers not exceeding 5 in U?

13. We have seen that the bit string for the set $\{1, 3, 5, 7, 9\}$ (with universal set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$) is 10 1010 1010.
What is the bit string for the complement of this set?

14. The bit strings for the sets $\{1, 2, 3, 4, 5\}$ and $\{1, 3, 5, 7, 9\}$ are 11 1110 0000 and 10 1010 1010, respectively. Use bit strings to find the union and intersection of these sets.

15. Let A, B, and C be sets. Show that

    (a) $(A \cap B) \subseteq A$
    (b) $(A - B) - C \subseteq A - C$
    (c) $A \cup (B - A) = A \cup B$
    (d) $(B - A) \cup (C - A) = (B \cup C) - A$

16. Find the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$.

17. Show that

    (a) $A \oplus B = (A \cup B) - (A \cap B)$
    (b) $A \oplus B = (A - B) \cup (A - B)$

18. What is the bit string corresponding to the difference of two sets?

19. What is the bit string corresponding to the symmetric difference of two sets?

20. Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? What is the composition of g and f ?

21. Prove or disprove that $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$ for all real numbers x and y.

22. Why is f not a function from R to R if

    (a) $f(x) = \frac{1}{x}$?
    (b) $f(x) = \sqrt{x}$
    (c) $f(x) = \pm\sqrt{(x^2 + 1)}$?

23. Find the domain and range of the following functions

    (a) the function that assigns the next largest integer to a positive integer
    (b) the function that assigns to a bit string the number of bits in the string
    (c) the function that assigns the number of bits left over when a bit string is split into bytes (which are blocks of 8 bits)
    (d) the function that assigns to each positive integer its largest decimal digit
    (e) the function that assigns to each positive integer the number of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 that do not appear as decimal digits of the integer

24. Determine whether each of these functions from Z to Z is one-to-one and onto.
    (a) $f(n) = n - 1$
    (b) $f(n) = n^2 - 1$
    (c) $f(n) = n^3$
    (d) $f(n) = \lceil \frac{n}{2} \rceil$
25. Determine whether each of these functions is a bijection from R to R.
    (a) $f(x) = -3x + 4$
    (b) $f(x) = x^5 + 1$
    (c) $f(x) = x^2 + 1$
    (d) $f(x) = x^3$
26. Prove that a strictly increasing function from R to itself is one-to-one.
27. If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer.
28. If f and $f \circ g$ are onto, does it follow that g is onto? Justify your answer.
29. Let $g(x) = \lfloor x \rfloor$. Find
    (a) $g^1(\{0\})$
    (b) $g^1(\{-1, 0, 1\})$
30. Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. How many different pairs of airlines can you choose on which to book a trip from NewYork to San Francisco via Denver, when you pick an airline for the flight to Denver and an airline for the continuation flight to San Francisco?
31. There are four major auto routes from Boston to Detroit and six from Detroit to Los Angeles. How many major auto routes are there from Boston to Los Angeles via Detroit?
32. How many bit strings are there of length six or less, not counting the empty string?
33. How many positive integers less than 1000
    (a) are divisible by 7?
    (b) are divisible by 7 but not by 11?
    (c) are divisible by both 7 and 11?
    (d) are divisible by either 7 or 11?
    (e) are divisible by exactly one of 7 and 11?
    (f) are divisible by neither 7 nor 11?
    (g) have distinct digits?
    (h) have distinct digits and are even?
34. How many strings of four decimal digits
    (a) do not contain the same digit twice?
    (b) end with an even digit?
    (c) have exactly three digits that are 9s?

35. How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?

36. How many functions are there from the set $\{1, 2, ..., n\}$, where n is a positive integer, to the set $\{0, 1\}$?

37. How many subsets of a set with 100 elements have more than one element?

38. Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.

39. Suppose that a computer science laboratory has 15 workstations and 10 servers.A cable can be used to directly connect a workstation to a server. For each server, only one direct connection to that server can be active at any time.We want to guarantee that at any time any set of 10 or fewer workstations can simultaneously access different servers via direct connections. Although we could do this by connecting every workstation directly to every server (using 150 connections), what is the minimum number of direct connections needed to achieve this goal?

40. During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games.

41. Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group.

42. A bowl contains 10 red balls and 10 blue balls.A woman selects balls at random without looking at them.

    (a) How many balls must she select to be sure of having at least three balls of the same color?

    (b) How many balls must she select to be sure of having at least three blue balls?

43. How many numbers must be selected from the set $\{1, 2, 3, 4, 5, 6\}$ to guarantee that at least one pair of these numbers add up to 7?

44. How many numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers add up to 16?

45. Acomputer network consists of six computers. Each computer is directly connected to at least one of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers.

46. Find the least number of cables required to connect eight computers to four printers to guarantee that for every choice of four of the eight computers, these four computers can directly access four different printers. Justify your answer.

47. Suppose that there are eight runners in a race. The winner receives a gold medal, the secondplace finisher receives a silver medal, and the third-place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there are no ties?

48. Suppose that a saleswoman has to visit eight different cities. She must begin her trip in a specified city, but she can visit the other seven cities in any order she wishes. How many possible orders can the saleswoman use when visiting these cities?

49. How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school?

50. A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission (assuming that all crew members have the same job)?

51. How many permutations of the letters ABCDEFGH contain

    (a) the string ED?

    (b) the string CDE?

    (c) the strings BA and FGH?

    (d) the strings AB, DE, and GH?

    (e) the strings CAB and BED?

    (f) the strings BCA and ABF?

52. A club has 25 members.

    (a) How many ways are there to choose four members of the club to serve on an executive committee?

    (b) How many ways are there to choose a president, vice president, secretary, and treasurer of the club, where no person can hold more than one office?

53. Find a formula for the number ofways to seat r of n people around a circular table, where seatings are considered the same if every person has the same two neighbors without regard to which side these neighbors are sitting on.

54. Find the coefficient of x^5y^8 in $(x+y)^{13}$

55. What is the coefficient of x^9 in $(2x)^{19}$

56. How many solutions does the equation $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2, and x_3 are nonnegative integers?

57. How many ways are there to distribute hands of 5 cards to each of four players from the standard deck of 52 cards?

58. How many different strings can be made from the letters in MISSISSIPPI, using all the letters?