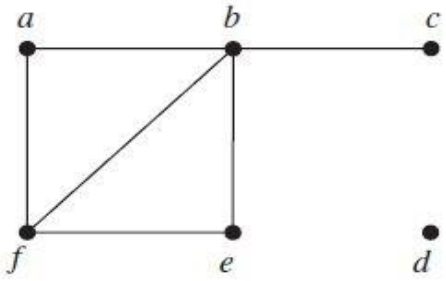
Discrete Mathematics. (3140708)

TUTORIAL 5: Graphs and Trees

1. From the following graphs find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

1.



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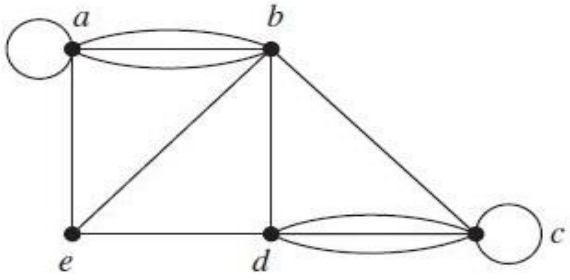

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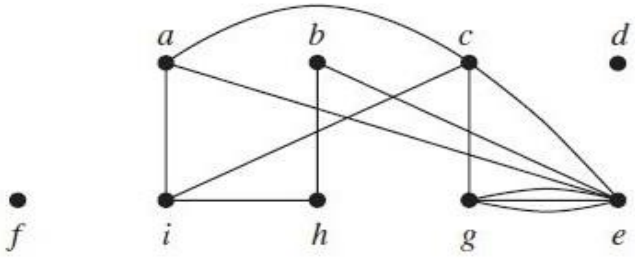
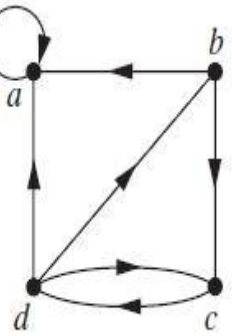


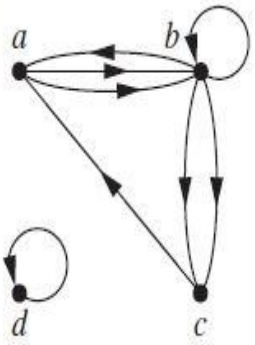
2. Can a simple graph exist with 15 vertices each of degree five?

3. From the graphs 7–9 determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.

7.



8.



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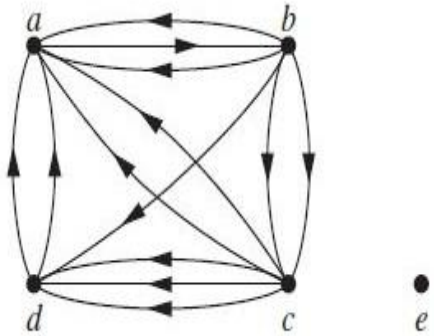


4. For each of the graphs 7–9 (Que. 3) determine the sum of the in-degrees of the vertices and the sum of the out-degrees of the vertices directly. Show that they are both equal to the number of edges in the graph.

5. Draw these graphs.

a) K_7 **b)** $K_{1,8}$ **c)** $K_{4,4}$
d) C_7 **e)** W_7 **f)** Q_4

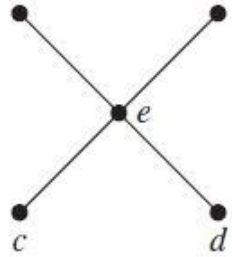
6. For which values of n are these graphs bipartite

a) K_n **b)** C_n **c)** W_n **d)** Q_n

7. For the graph **1** in Que.1 find the subgraph induced by the vertices a, b, c, and f.

8. In graphs 21–24 determine whether the graph is bipartite.

21.



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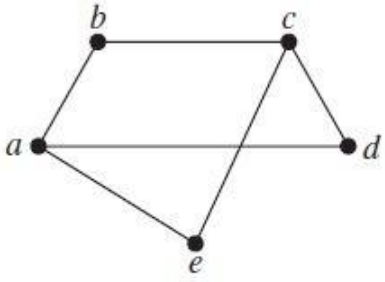

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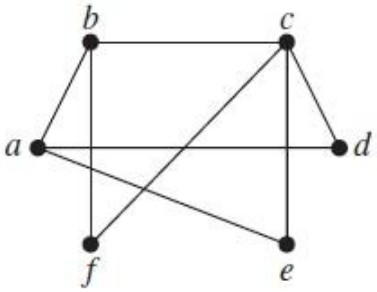

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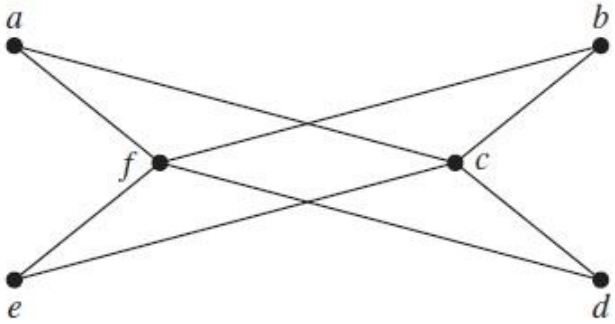
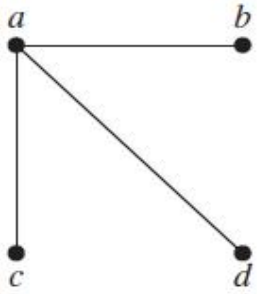


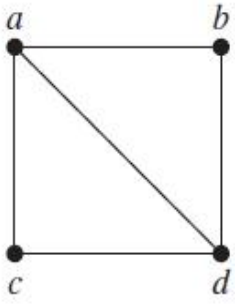
9. How many subgraphs with at least one vertex does K_3 have?

10. Draw all subgraphs of this graph.



11. Represent the graphs 1-4 with an adjacency matrix.

1.



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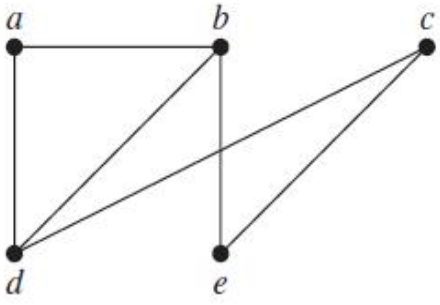

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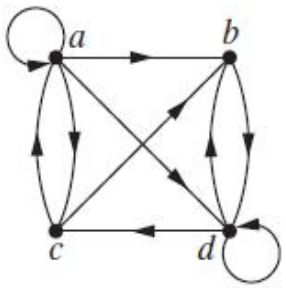

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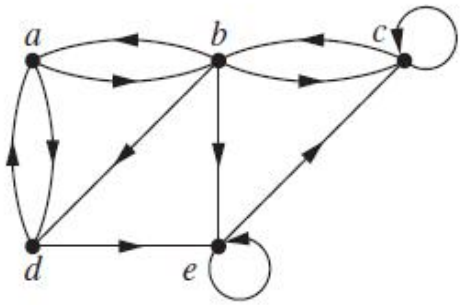
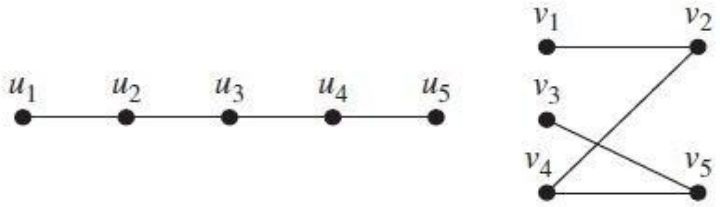


12. Draw a graph with the given adjacency matrix.

1. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **2.** $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

13. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

1.



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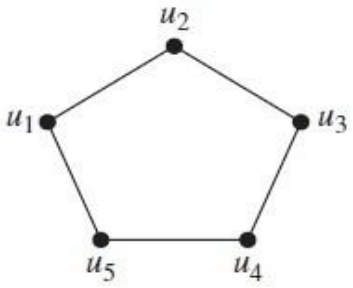

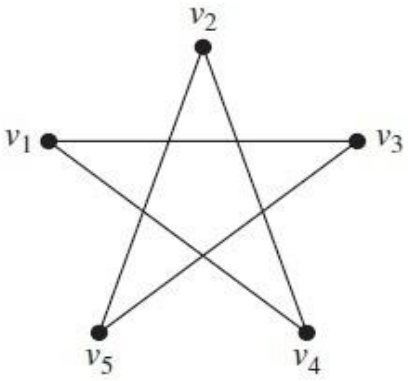

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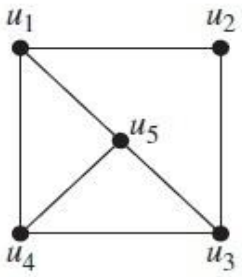

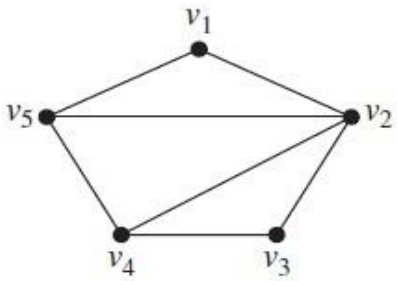

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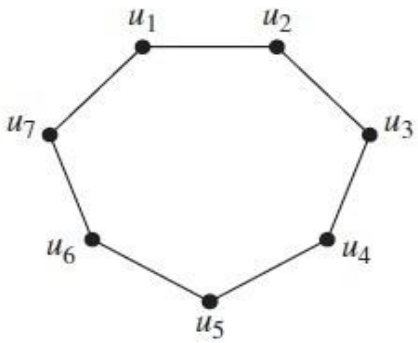
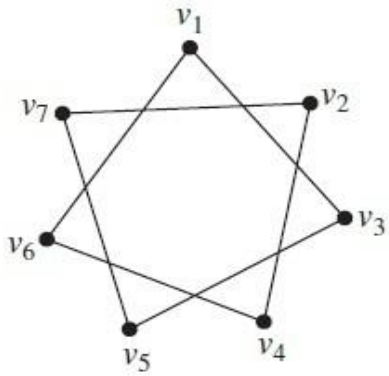
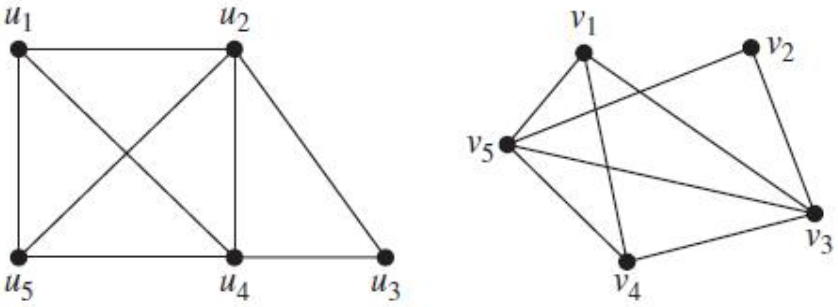




5.


u_1
u_2
u_5
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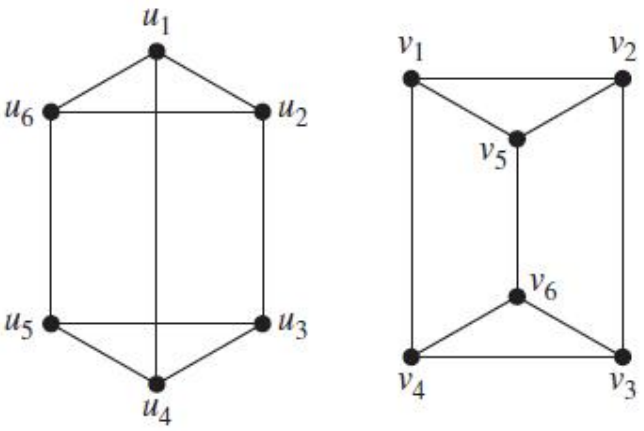
u_3
v_1
v_2
v_5
v_3
v_4

6.


u_1
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v_1
v_2
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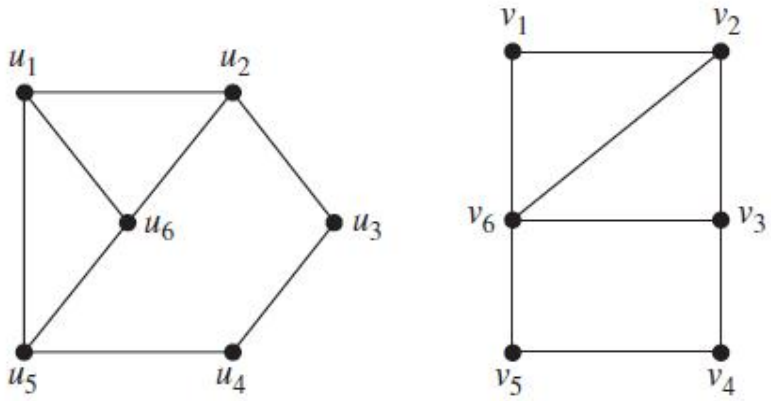
7.


u_1
u_2
u_6
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v_1
v_2
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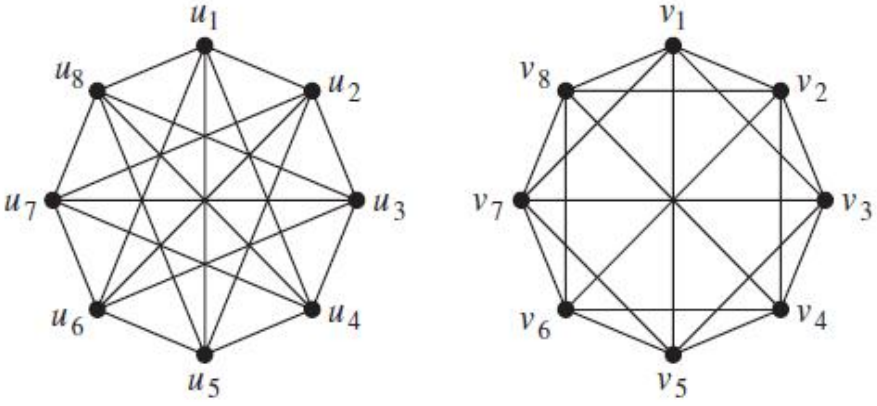
8.


u_1
u_8
u_2
u_7
u_3
u_6
u_4
u_5
v_1
v_8
v_2
v_7
v_3
v_6
v_4
v_5

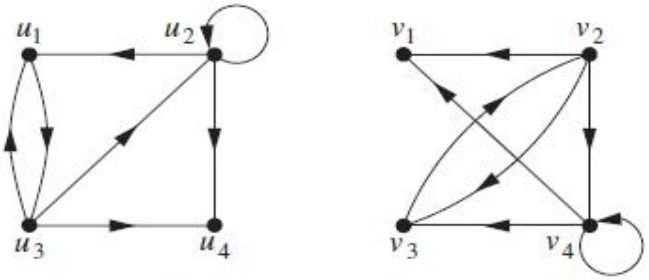
14. Draw an undirected graph represented by the given adjacency matrices.

1. $\begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 4 \\ 2 & 4 & 0 \end{bmatrix}$ **2.** $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

15. How many nonisomorphic simple graphs are there with five vertices and three edges?

16. Determine whether the given pair of directed graphs are isomorphic

1.



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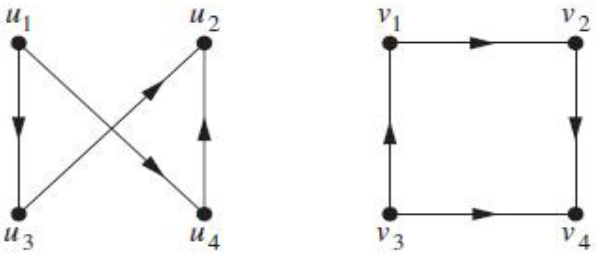
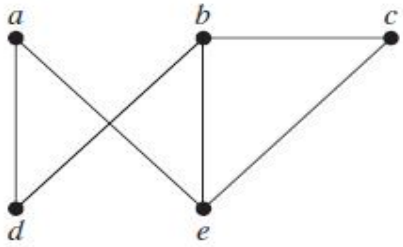


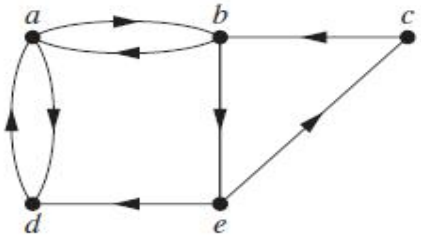
17. Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?

a) *a, e, b, c, b* **b)** *a, e, a, d, b, c, a* **c)** *e, b, a, d, b, e* **d)** *c, b, d, a, e, c*



18. Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?

a) *a, b, e, c, b* **b)** *a, d, a, d, a* **c)** *a, d, b, e, a* **d)** *a, b, e, c, b, d, a*

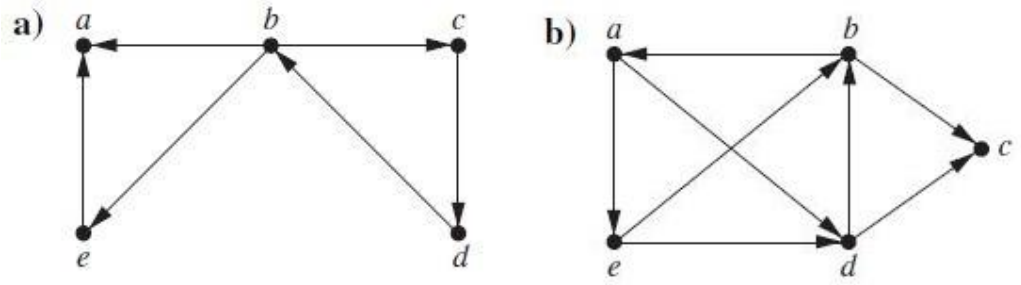


19. Determine whether the given graph is connected.

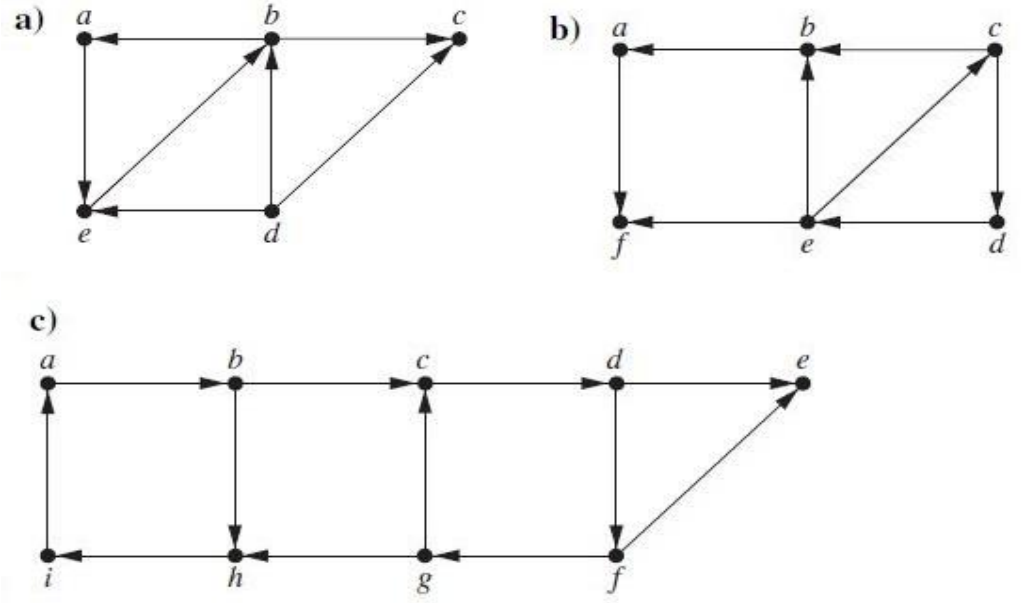
1.

2.

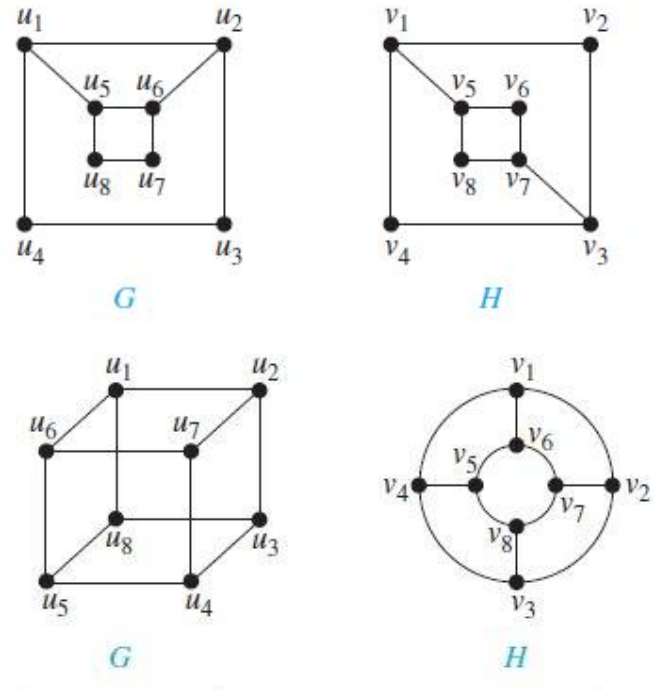
20. Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected.



21. Find the strongly connected components of each of these graphs.

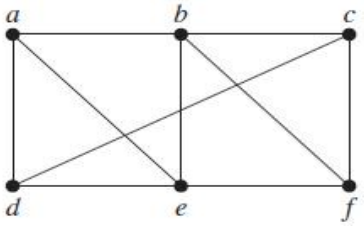


22. Use paths either to show that these graphs are not isomorphic or to find an isomorphism between these graphs.

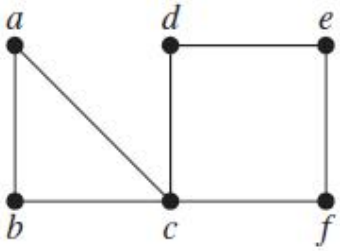


23. Find the number of paths between *c* and *d* in the graph of length

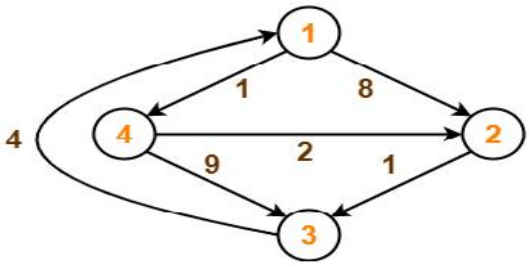
a) 2. **B)** 3. **C)** 4. **D)** 5. **E)** 6. **F)** 7.



24. How many paths of length four are there from a to e in the simple graph.

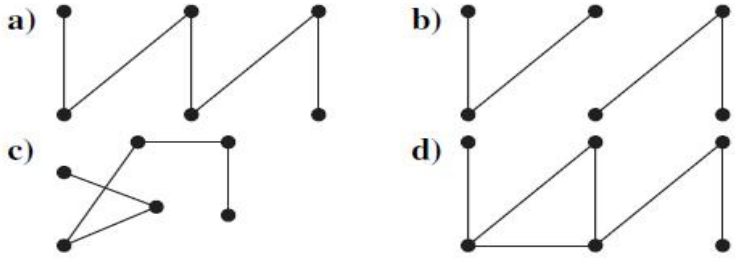


25. Consider the following directed weighted graph,

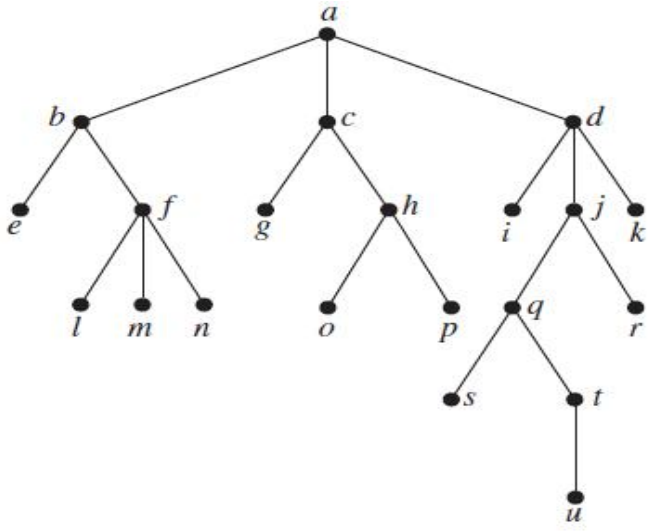


Using Floyd Warshall Algorithm, find the shortest path distance between every pair of vertices

26. Which of these graphs are trees?



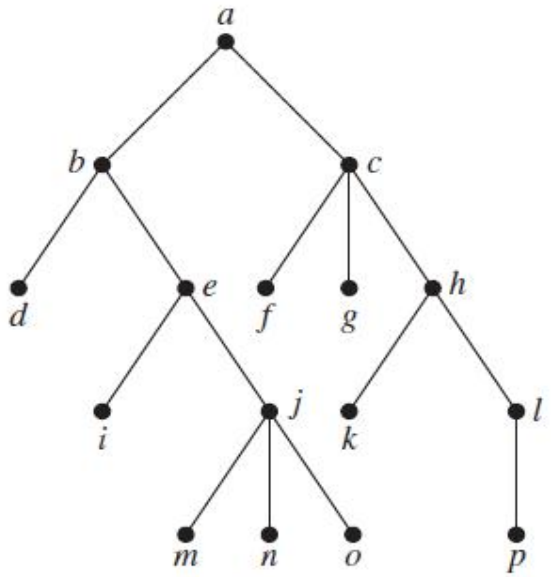
27. Answer these questions about the rooted tree illustrated



1. Which vertex is the root?
2. Which vertices are internal?
3. Which vertices are leaves?
4. Which vertices are children of *j*?
5. Which vertex is the parent of *h*?
6. Which vertices are siblings of *o*?
7. Which vertices are descendants of *b*?
8. Is the rooted tree a full *m*-ary tree for some positive integer *m*?
9. Draw the sub-tree of the tree that is rooted at *a* and *c*.
10. What is the level of each vertex of the rooted tree.
11. What is the level of each vertex of the rooted tree.

28. Construct a complete binary tree of height 4 and a complete 3-ary tree of height 3.

29. Answer these questions about the rooted tree illustrated



1. In which order are the vertices of the ordered rooted tree in visited using an inorder traversal?
2. In which order are the vertices of the ordered rooted tree in visited using an postorder traversal?

30. Determine the order in which a preorder traversal visits the vertices of the given ordered rooted tree.