

GUJARAT TECHNOLOGICAL UNIVERSITY**BE- SEMESTER-III (NEW) EXAMINATION – WINTER 2020****Subject Code:3130908****Date:09/03/2021****Subject Name:Applied Mathematics for Electrical Engineering****Time:10:30 AM TO 12:30 PM****Total Marks:56****Instructions:**

1. Attempt any FOUR questions out of EIGHT questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Find a root of the equation $x^4 - x - 10 = 0$ using Bisection method. Perform only four iterations. **03**
- (b) Use Newton's divided difference formula to find $f(x)$ from the following data: **04**
- | | | | | |
|-----|-----|-----|----|----|
| x | 3 | 7 | 9 | 11 |
| y | 168 | 120 | 72 | 48 |
- Hence evaluate y for $x = 6$.
- (c) (i) Use Trapezoidal rule to evaluate $\int_0^1 x^2 dx$ considering five subintervals. **03**
- (ii) Apply Runge-Kutta fourth order method to find an approximate value of y when $x = 0.2$ given that **04**
- $$\frac{dy}{dx} = y - \frac{2x}{y}, \quad y(0) = 1, \quad h = 0.2.$$
- Q.2** (a) Find the mean, median and standard deviation for the following data: **03**
- 48, 43, 65, 57, 31, 60, 37, 48, 59, 78.
- (b) If the probability density of a random variable is given by **04**
- $$f(x) = \begin{cases} k(1 - x^2), & \text{for } 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$
- find k . Also find the probabilities that a random variable having this probability density will take on a value (a) between 0.1 and 0.2 (b) greater than 0.5.
- (c) (i) Find a root of the equation $xe^x - \cos x = 0$ in the interval $(0, 1)$ using Newton-Raphson Method correct up to $\epsilon_a < 1\%$. Take $x_0 = 0.5$. **03**
- (ii) Find a real root of the equation $x^3 + x^2 - 100 = 0$ correct to two decimal places using Fixed Point Iteration method. **04**
- Q.3** (a) Use Newton's backward interpolation formula to find the value of $f(175)$ from the following table: **03**
- | | | | | | |
|--------|------|------|------|------|-------|
| x | 140 | 150 | 160 | 170 | 180 |
| $f(x)$ | 3685 | 4845 | 6302 | 8076 | 10225 |
- (b) If $y(1) = -3, y(3) = 9, y(4) = 30, y(6) = 132$, find the Lagrange's interpolation polynomial that takes the same values as y at the given point. **04**
- (c) The following show the gain in reading speed of 8 students in a speed-reading program, and the number of weeks they have been in the program: **07**
- | | | | | | | | | |
|--------------|----|-----|----|-----|-----|-----|----|-----|
| No. of weeks | 3 | 5 | 2 | 8 | 6 | 9 | 3 | 4 |
| Speed gain | 86 | 118 | 49 | 193 | 164 | 232 | 73 | 109 |
- Fit a straight line by the method of least squares.
- Q.4** (a) The population (in thousands) of a town is given below. Estimate the population for the year 1975 using interpolation. **03**
- | | | | | | |
|------------|------|------|------|------|------|
| Year | 1971 | 1981 | 1991 | 2001 | 2011 |
| Population | 46 | 66 | 81 | 93 | 101 |
- (b) In usual notations, prove the following identities: **04**

$$(i) 1 + \mu^2 \delta^2 = \left(1 + \frac{1}{2} \delta^2\right)^2 \quad (ii) \mu\delta = \frac{1}{2} \Delta E^{-1} + \frac{1}{2} \Delta.$$

- (c) Fit a parabola $y = a + bx + cx^2$ to the following data: 07
- | | | | | | |
|-----|--------|---------|---------|---------|----------|
| x | 1 | 2 | 3 | 4 | 6 |
| y | 9.7468 | 24.4451 | 47.9318 | 78.4660 | 164.4186 |

- Q.5** (a) Find the value of $y(0.4)$ from the following differential equation with the given initial condition by Euler's method: 03

$$\frac{dy}{dx} = \log(x + y), \quad y(0) = 2, \quad h = 0.1.$$

- (b) Evaluate $\int_2^4 (x^2 + 2x) dx$ by using Gauss' quadrature formula with $n = 3$. 04

- (c) (i) An assembly plant receives its voltage regulators from three different suppliers, 60 % from supplier B_1 , 30 % from supplier B_2 , and 10 % from supplier B_3 . If 95 % of the voltage regulators from B_1 , 80 % of those from B_2 , and 65 % of those from B_3 perform according to specifications, what is the probability that any one voltage regulator received by the plant will perform according to specifications? Also, find the probability that a particular voltage regulator, known to perform according to specifications, came from supplier B_3 . 03

- (ii) Find the missing frequencies f_1 and f_2 if the mean of the following frequency distribution of 100 families (f) is 30.4: 04

x	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
f	10	f_1	25	30	f_2	10

- Q.6** (a) Find, by Taylor's series method, the value of y at $x = 0.1$ to five places of decimals from 03

$$\frac{dy}{dx} = x^2 y - 1, \quad y(0) = 1.$$

- (b) Evaluate $\int_{0.2}^{1.4} (2 + x \log x - \cos x) dx$ with $h = 0.2$ by Simpson's one-third rule and Simpson's three-eighth rule. 04

- (c) (i) The probability that an integrated circuit chip will have defective etching is 0.12, the probability that it will have a crack defect is 0.29, and the probability that it has both defects is 0.07. What is the probability that a newly manufactured chip will have neither defect? 03

- (ii) A standard cell whose voltage is known to be 1.10 volts was used to test the accuracy of two volt meters A and B . Ten independent readings of the voltage of the cells were taken with the two volt meters as per the following data. Which of these two is more reliable? 04

A	1.11	1.15	1.14	1.10	1.09	1.11	1.12	1.15	1.13	1.14
B	1.12	1.06	1.02	1.08	1.11	1.05	1.56	1.03	1.04	1.06

- Q.7** (a) Find the mode for the following frequency distribution: 03

Class	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
f	20	30	25	16	12

- (b) Calculate the coefficient of skewness based on the Method of Moments from the following data: 04

Class	0 – 4	5 – 9	10 – 14	15 – 19	20 – 24
Frequency	7	12	15	10	6

- (c) (i) For a random variable X , if $E(3X - 5) = 16$ and $E(X^2) = 58$, find the standard deviation of X . 03

- (ii) If the events A and B are independent, then show that the events A and B' are also independent. 04

- Q.8 (a)** Calculate the mean and standard deviation from the following data: **03**
- | | | | | | | | |
|-----------|-------|-------|-------|-------|-------|-------|-------|
| Value | 90-99 | 80-89 | 70-79 | 60-69 | 50-59 | 40-49 | 30-39 |
| Frequency | 2 | 12 | 22 | 20 | 14 | 4 | 1 |
- (b)** Find the mean deviation from median for the following data: **04**
- | | | | | | |
|----------|--------|---------|---------|---------|---------|
| Marks | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 |
| Students | 8 | 11 | 15 | 9 | 7 |
- (c) (i)** Three students A, B and C are running in a race. A and B have the same probability of winning and each is twice as likely to win as C . Find the probability that B or C wins. **03**
- (ii)** The quantities of milk (in liters) produced by a dairy farm on ten consecutive days are shown below: **04**
- 218.2, 199.7, 207.3, 185.4, 213.7, 184.7, 179.5, 194.4, 224.3, 203.5.
- Evaluate the mean and the first four central moments of the milk yield data (in litres) of dairy farm.
