

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER- IV EXAMINATION – SUMMER 2020****Subject Code: 3140610****Date:02/11/2020****Subject Name: Complex Variables and Partial Differential Equations****Time: 10:30 AM TO 01:00 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

| | Mark s |
|--|-------------------|
| Q.1 (a) Show that the function $u = x^2 - y^2$ is harmonic and find the corresponding analytic function. | 03 |
| (b) Find the fourth roots of -1 . | 04 |
| (c) (i) Find the image of the infinite strip $0 \leq x \leq 1$ under the transformation $w = iz + 1$. Sketch the region. | 03 |
| (ii) Write the function $f(z) = z + \frac{1}{z}$ in the form $f(z) = u(r, \theta) + iv(r, \theta)$. | 04 |
| Q.2 (a) Evaluate $\int_C (x^2 + ixy) dz$ from $(1,1)$ to $(2,4)$ along the curve $x = t, y = t^2$. | 03 |
| (b) Find the bilinear transformation which transforms $z = 2, 1, 0$ into $w = 1, 0, i$ | 04 |
| (c) (i) Evaluate $\oint_C \frac{dz}{z^2 + 1}$, where C is $ z + i = 1$, counter clockwise. | 03 |
| (ii) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} z^n$. | 04 |
| OR | |
| (c) (i) Find the roots of the equation $z^2 + 2iz + (2 - 4i) = 0$. | 03 |
| (ii) Find the roots of $\log z = i \frac{\pi}{2}$. | 04 |
| Q.3 (a) Find $\oint_C \left(\frac{3}{z-i} - \frac{6}{(z-i)^2} \right) dz$, where $C : z = 2$. | 03 |
| (b) Find the residues of $f(z) = \frac{1}{(z-1)^2(z-3)}$, has a pole at $z = 3$ and a pole of order 2 at $z = 1$. | 04 |
| (c) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series for the regions (i) $ z < 1$, (ii) $1 < z < 3$, (iii) $ z > 3$. | 07 |
| OR | |
| Q.3 (a) Evaluate $\oint_C \frac{e^z}{z+i} dz$, where $C : z-1 = 1$. | 03 |
| (b) Evaluate by using Cauchy's residue theorem $\int_C \frac{5z-2}{z(z-1)} dz ; z = 2$. | 04 |

- (c) Expand $f(z) = \frac{1}{(z+1)(z-2)}$ in Laurent's series for the regions (i) $|z| < 1$, **07**
(ii) $1 < |z| < 2$, (iii) $|z| > 2$.
- Q.4** (a) Solve $yq - xp = z$. **03**
(b) Derive partial differential equation by eliminating the arbitrary constants a and b from $z = (x^2 + a)(y^2 + b)$. **04**
(c) (i) Solve the p.d.e. $r - 3as + 2a^2t = 0$. **03**
(ii) Find the complete integral of $p(1+q) = qz$. **04**
- OR**
- Q.4** (a) Find the solution of $(y-z)p + (z-x)q = x-y$. **03**
(b) Form the partial differential equation by eliminating the arbitrary function from $\phi(x+y+z, x^2+y^2+z^2) = 0$. **04**
(c) (i) Solve the p.d.e. $s + p - q = z + xy$. **03**
(ii) Solve by Charpit's method $q = 3p^2$. **04**
- Q.5** (a) Solve $(D^2 + 2DD' + D'^2)z = e^{2x+3y}$ **03**
(b) Solve the p.d.e. $u_x = 4u_y, u(0, y) = 8e^{-3y}$ using the method of separation of variables. **04**
(c) Find the solution of the wave equation $u_{tt} = c^2 u_{xx}, 0 \leq x \leq L$ with the conditions $u(0, t) = u(L, t) = 0; t > 0, u(x, 0) = \frac{\pi x}{L}, u_t(x, 0) = 0; 0 \leq x \leq L$. **07**
- OR**
- Q.5** (a) Solve the p.d.e. $r + s + q - z = 0$. **03**
(b) Solve $xu_x - 2yu_y = 0$ using the method of separation of variables. **04**
(c) Find the solution of $u_t = c^2 u_{xx}$ together with the initial and boundary conditions $u(0, t) = u(l, t) = 0; t \geq 0$ and $u(x, 0) = \sin \frac{\pi x}{l}; 0 \leq x \leq l$. **07**