Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–IV (NEW) EXAMINATION – WINTER 2020

Subject Code:3140610 **Date:17/02/2021**

Subject Name:Complex Variables and Partial Differential Equations

Time:02:30 PM TO 04:30 PM **Total Marks:56**

Instructions:

1. **Attempt any FOUR questions out of EIGHT questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Show that the function $u = x^3 - 3xy^2$ is harmonic.	**03**
	(b)	Find the fifth root of unity.	**04**
	(c)	(i) Determine and sketch the image of $\|z\| = 1$ under the transformation $w = z + i.$	**03**
		(ii) Find the real and imaginary parts of $f(z) = \dfrac{3i}{2+3i}$.	**04**
Q.2	**(a)**	Evaluate $\int_C (x^2 - iy^2)dz$ along the parabola $y = 2x^2$ from (1,2) to (2,8)	**03**
	(b)	Find the bilinear transformation that maps the points $z = \infty, i, 0$ into the points $w = 0, i, \infty$ respectively.	**04**
	(c)	(i) Evaluate $\oint_C \dfrac{\sin 3z}{z + \pi/2} dz$ where C is the circle $\|z\| = 5.$	**03**
		(ii) For which values of *z* does the series $\sum_{n=0}^{\infty} n! z^n$ convergent?	**04**
Q.3	**(a)**	Evaluate $\oint_C \dfrac{e^{-z}}{z+1} dz$ where C is the circle $\|z\| = 1/2.$	**03**
	(b)	Find the residue *Res(f(z), -1)* for $f(z) = \dfrac{z^2 - 2z}{(z+1)^2(z^2+4)}$.	**04**
	(c)	Expand $f(z) = \dfrac{1}{(z+2)(z+4)}$ in the region $(i)\|z\| < 2$, $(ii) 2 < \|z\| < 4$, $(iii)\|z\| > 4.$	**07**
Q.4	**(a)**	Evaluate $\oint_C \dfrac{2z+6}{z^2+4} dz$ where C is $\|z - i\| = 2.$	**03**
	(b)	Using Cauchy's Residue Theorem evaluate $\int_C \dfrac{e^{2z}}{(z+1)^3} dz$, where C is the ellipse $4x^2 + 9y^2 = 16.$	**04**

	(c)	Expand $\frac{1}{z(z^2-3z+2)}$ about $z = 0$ for the region $(i) 0<\lvert z\rvert<1$, $(ii) 1<\lvert z\rvert<2$, $(iii) \lvert z\rvert<2$.	**07**
Q.5	**(a)**	Solve $y^2 p - xyq = x(z-2y)$.	**03**
	(b)	Derive p.d.e. by eliminating a and b from $z=(x-a)^2+(y-b)^2$.	**04**
	(c)	(i) Solve $(D^3-3D^2D'+2D'^3)z=0$.	**03**
		(ii) Find the complete integral of $p^2=qz$.	**04**
Q.6	**(a)**	Solve $x^2p+y^2q=z^2$.	**03**
	(b)	Form a p.d.e. by eliminating the arbitrary function from $z=f(x^2-y^2)$.	**04**
	(c)	(i) Solve $(D-D'-1)(D-D'-2)z=e^{2x-y}+x$.	**03**
		(ii) Solve $(p^2+q^2)y=qz$ by Charpit's method.	**04**
Q.7	**(a)**	Solve $2r-5s+2t=24(y-x)$.	**03**
	(b)	Solve the p.d.e. $u_x+u_y=2(x+y)u$.	**04**
	(c)	A tightly stretched string with fixed end points $x = 0$, $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $\lambda x(l-x)$, find the displacement $u(x,t)$.	**07**
Q.8	**(a)**	Solve $(D^2-D'^2+D-D')z=0$.	**03**
	(b)	Solve the p.d.e. $u_{xx}=16u_y$.	**04**
	(c)	Find the solution of one dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ satisfying the conditions $u(0.t)=u(\pi,t)=0$ for $t > 0$ and $u(x,0)=\pi-x, 0<x<\pi$.	**07**

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