HASMUKH GOSWAMI COLLEGE OF ENGINEERING, VAHELAL

Complex Variables and PDE (3130005)

Tutorial-1 (Complex Numbers & Analytic Functions)

1. Find real and imaginary part of $f(z) = z^2 + 3z$.
2. Find the value of $\operatorname{Re}\left(f(z)\right)$ and $\operatorname{Im}\left(f(z)\right)$ at the indicated points where $f(z) = \dfrac{1}{1-z}$ at $7+2i$.
3. Write the function $f(z) = z + \dfrac{1}{z}$ in $f(z) = u(r,\text{¯}) + iv(r,\text{¯})$ form.
4. Prove that (i) $\tan^{-1} z = \dfrac{i}{z}\log\left(\dfrac{i+z}{i-z}\right)$ (ii) $\sin^{-1} z = -i\log(iz + \sqrt{1-z^2})$
5. Find the value of $(-i)^i$
6. Find the principle argument of $z = \dfrac{-2}{1+i\sqrt{3}}$
7. Find and plot all the roots of $(a)\ (1+i)^{\frac{1}{3}}$ $(b)\ \sqrt[3]{8i}$
8. Show the set of values of $\log(i^2)$ is not the same as the set of values $2\log i$
9. Is $Arg(z_1 \cdot z_2) = Arg(z_1) + Arg(z_2)$? Justify
10. For all principle branch show that $\log(i^3) \neq 3\log(i)$
11. Find real and imaginary part of $(-1-i)^7 + (-1+i)^7$
12. Define $\log\ (x+iy)$. Determine $\log(1-i)$
13. Show that if c is any n^{th}root of unity other than unity itself then $1 + c + c^2 + c^3 + \ldots + c^{n-1} = 0$
14. Solve the equation $z^2 - (5+i)z + (8+i) = 0$
15. Define domain. Is the set $|z-1+2i| \leq 2$ domain?
16. Sketch $S = \{z / -1 < \operatorname{Im}(z) < 2\}$, Is it connected?
17. Prove $\lim\limits_{z\to 1} \dfrac{i\bar{z}}{3} = \dfrac{i}{3}$ by defination
18. Use the £-¯ Definition of limit to show that $\lim\limits_{z\to 3i} = (3x+iy)^2 = 9i,\ where\ z = x+iy$
19. Prove that $|\exp(-2z)| < 1 \Leftrightarrow \operatorname{Re} z > 0$
20. Show that $\cos(iz) = \overline{\cos(i\bar{z})}, \forall z$
21. Find the values of the derivative of $\dfrac{z-i}{z+i}$ at i
22. Show that $f(z) = z \cdot \operatorname{Im}(z)$ is differntiable only at z=0 and $f'(0) = 0$
23. Find the principle value of $\left[\dfrac{e}{2}\left(-1-i\sqrt{3}\right)\right]^{3®i}$
24. Find out (and give reason) Whether $f(z)$ is continuous at $z = 0$

    if, $f(z) = \dfrac{\operatorname{Re}(z^2)}{|z|}, z \neq 0$

    $= 0 \quad , z = 0$

25. (1) State and prove cauchy-riemann condition for a function $f(z)=u(x,y)+iv(x,y)$

(2) Let a function $f(z)$ be analytic in a domain D. Prove that $f(z)$ must be constant in D in each of the following case

(a) if $f(z)$ is real valued for all z in D

(b) if $f(z)$ is analytic in D.

26. The function $f(z)=\begin{cases}\dfrac{\bar{z}}{z}, & z\neq 0\\ 0, & z=0\end{cases}$ Satisfies cauchy-Riemann equation but $f'(0)$ fails to exist.

27. Determine Harmonic function, Show that $u = x\sin x\ \cosh y - y\cos x\ \sinh y$ is harmonic.

28. Show that if $f(z)$ is analytic in a domain D and $|f(z)| = k =$ constant in D, then $f(z) =$ constant in D

29. Chek whether the function $f(z)=\sin z$ is analytic or not. If analytic find its derivative

30. Chek whether the following function are analytic or not

(1) $f(z)=z^{\frac{5}{2}}$ (2) $f(z)=\bar{z}$ (3) $f(z)=e^{\bar{z}}$ (4) $f(z)=2x+ixy^2$

31. Show that $u(x,y)=x^2-y^2$ is harmonic. Find the corresponding analytic function $f(z)=u(x,y)+iv(x,y)$

32. Determine a and b suc that $u=ax^3+bxy$ is harmonic in some doamin and find a harmonic conjugate

33. Show that $u(x,y)=2x-x^3+3xy^2$ is harmonic in some domain and find harmonic conjugate of $u(x,y)$

34. Show that $u(x,y)=e^{x^2-y^2}\cos 2xy$ is harmonic everywhere and find a conjugate harmonic for u(x,y)

35. Show that $u(x,y)=x^2-y^2+x$ is harmonic.
Find the corresponding analytic function $f(z)=u(x,y)+iv(x,y)$

36. Find the analytic function $f(z)=u+iv$ if $u-v=(x-y)(x^2+4xy+y^2)$

37. Determine an analytic function $f(z)=u+iv$ if
$u+v=e^x\left[x(\cos y+\sin y)+y(\cos y-\sin y)\right]$

Note: All the questions were asked in GTU Examination.