HASMUKH GOSWAMI COLLEGE OF ENGINEERING, VAHELAL

Complex Variables and PDE (3130005)

Tutorial-2 Conformal Mapping & Its Applications

1. Determine and sketch the image of $|z|=1$ under the transformation $w=z+i$.
2. Find the image of the semi-infinite strip $x>0,\ \ 0<y<2$ when $w=iz+1$. Sketch the strip and its image.
3. Find the bilinear transformation that maps the points $z_1=1, z_2=-1,\ z_3=\infty$ onto $w_1=1+i, w_2=1-i, w_3=1$ respectively. Also find its fixed points.
4. Describe geometrically the transformation $w=\frac{1}{z}$. State why it transforms circles and lines into circles and lines.
5. Find the image of infinite strips
   a. $\frac{1}{4}\le y\le\frac{1}{2}$
   b. $0\le y\le\frac{1}{2}$ Under the transformation $w=\frac{1}{z}$. Also show that the regions graphically.
6. Define fixed point of bilinear transformation. Find fixed point of $w=\frac{z+1}{z}$. Verify your result.
7. Find and graph the strip $1<x<z$ under the mapping $w=\frac{1}{z}$.
8. Find and sketch the region $x\ge 1$under the transformation $w=\frac{1}{z}$.
9. Explain Translation, Rotation and Magnification transformation. Find the image of the $|z-1|=1$ under the transformation $w=\frac{1}{z}$.
10. Show that the transformation $W=\sin z$ maps the top half $(y>0)$ of the line $x=c_1$ $\left(-\frac{®}{2}<c_1<0\right)$ in a one to one manner onto the half $(v>0)$ of the left hand branch of the hyperbola $\frac{u^2}{\sin^2 c_1}-\frac{v^2}{\cos^2 c_1}=1.$
11. Determine the angle of rotation at the points $z=2+i$ when the transformation is $w=z^2$, and illustrate it for some particular curve. Show that the scale factor of the transformation at that point is $2\sqrt{5}$.
12. Determine the image of infinite of the regions under $w=\frac{1}{z}$. **(a)** $x>1, y>0$ **(b)** $0<y<\frac{1}{2c}$.
13. Show that under the transformation $w=\frac{1}{z}$ circle $x^2+y^2-6x=0$ is transformed into a straight line in the W-plane.
14. Find the bilinear transformation that maps the points $z_1=1, z_2=i,\ z_3=-1$ onto $w_1=-1, w_2=0, w_3=1$ respectively. Find the image of $|z|<1$ under this transformation.
15. Define Mobius transformation. Determine the Mobius transformation that maps $z=0,1,\infty$ on to $w=-1,-i,1$ respectively.

16. Attempt following.

- **a.** Find the bilinear transformation which maps the points $z = 1, i, -1$ on to $w = i, 0, -i$.
- **b.** Hence find the image of $|z| < 1$
- **c.** Find also invariant points of this transformation.

17. Define a Linear Fractional Transformation (Mobius transformation). Find the bilinear transformation that maps the points $z_1 = -1, z_2 = 0,\ z_3 = 1$ onto $w_1 = -i, w_2 = 1, w_3 = i$ respectively. Also find w for $z = \infty.$

18. Show that when $\operatorname{Im} z_0 < 0,$ the transformation $w = e^{i® \frac{z-z_0}{z-\bar{z}_0}}$ maps the lower half plane $\operatorname{Im} z \le 0$ onto unit disk $|w| \le 1.$

19. Find the bilinear transformation that maps the points $z_1 = 0, z_2 = \infty,\ z_3 = i$ onto $w_1 = \infty, w_2 = 1, w_3 = 0$ respectively.

20. Define Mobius transformation. Also find the image of the circle $|z| = 1$ in the w-plane under the Mobius transformation $w = f(z) = \dfrac{z-i}{1-iz}$. Also find the fixed points of f.

21. Find the image of the region bounded by $1 \le r \le 2$ and $\frac{®}{6} \le ¯ \le \frac{®}{3}$ in the z-plane under the transformation $w = z^2$. Show the region graphically.

Note: All the questions were asked in GTU Examination.