HASMUKH GOSWAMI COLLEGE OF ENGINEERING, VAHELAL

Probability and Statistics (3130006)

TUTORIAL 5: RANDOM VARIABLES

1. Define: Discrete Probability Distribution, Discrete Distribution Function, Continuous Probability Distribution, Continuous Distribution Function.

2. A random variable X has the following Probability function:

X	0	1	2	3	4
P(X=x)	K	3k	5k	7k	9k

Find (i) the value of k, (ii) $P(X<3)$, $P(X\geq 3)$, $P(0<X<4)$, and (iii) Distribution function of X.

Ans: (i) 1/25, (ii) 9/25, 16/25, 3/5, and (iii) F(0)=1/25, F(1)=4/25, F(2)=9/25, F(3)=16/25, F(4)=1

3. A discrete random variable X has the following distribution function:

$$F(x)=\begin{cases} 0, & x<1 \\ \frac{1}{3}, & 1\leq x<4 \\ \frac{1}{2}, & 4\leq x<6 \\ \frac{5}{6}, & 6\leq x<10 \\ 1, & x\geq 10 \end{cases}$$

Find $(i)\ P(2<X\leq 6), (ii)\ P(X=5), (iii)\ P(X\leq 6), (iv)\ P(x=4)$.

Ans: (i) 1/2, (ii) 0, (iii) 5/6, (iv) 1/6

4. A discrete random variable can take all integer values from 1 to k each with the probability of $\frac{1}{k}$.

Show that its Mean and Variance are $\frac{k+1}{2}$ & $\frac{k^2-1}{12}$.

5. For the following Probability distribution:

X	-3	-2	-1	0	1	2	3
P(X=x)	0.001	0.01	0.1	?	0.1	0.01	0.001

Find (i) Missing Probability, (ii) Mean, and (iii) Variance. **Ans: (i) 0.778, (ii) 0.2, (iii) 0.258**

6. Let X denotes the minimum of two numbers that appear when a pair of fair dice is thrown once. Determine (i) Probability Distribution, (ii) Expectation, and (iii) Variance. **Ans: (ii) 2.5278, (iii) 1.9713, and (i)**

X	**1**	**2**	**3**	**4**	**5**	**6**
P(X=x)	**11/36**	**9/36**	**7/36**	**5/36**	**3/36**	**1/36**

7. Find the value of k such that $f(x)$ is a probability density function. Find also, $P(X\leq 1.5)$.

$$f(x)=\begin{cases} kx & 0\leq x\leq 1 \\ k & 1\leq x\leq 2 \\ k(3-x) & 2\leq x\leq 3 \end{cases}$$

Ans: k=1/2, 1/2

8. Let X be a continuous random variable with pdf $f(x)=kx(1-x),\ 0\leq x\leq 1$. Find k and determine a number b such that $P(X\leq b)=P(X\geq b)$. **Ans: k=6, b=1/2**

9. Verify that the function $F(x)=\begin{cases} 0 & x<0 \\ 1-e^{-\frac{x}{4}} & x\geq 0 \end{cases}$ is a distribution function. Also, find

$P(X\leq 4), P(X\geq 8), P(4\leq X\leq 8)$. **Ans: (e-1)/2, $1/e^2$, $(e-1)/e^2$**

10. A continuous random variable has probability density function $f(x)=6(x-x^2),\ 0\leq x\leq 1$. Find the (i) Mean, (ii) Variance, (iii) Median, (iv) Mode. **Ans: (i) 1/2, (ii) 1/20, (iii) 1/2, (iv) 1/2**

11. The pdf of a continuous random variable X is $f(x)=\frac{1}{2}e^{-|x|}$. Find Cumulative distribution function.

Ans: $F(x)=\begin{cases} \frac{1}{2}e^{x} & x\le 0 \\ 1-\frac{1}{2}e^{-x} & x\ge 0 \end{cases}$

12. Let X be a random variable with E(X)=10 and Var(X)=25. Find the positive values of a and b such that Y=aX-b has an expectation of 0 and a variance of 1. **Ans: a=1/5, b=2**

13. A sampling a large number of parts manufactured by a machine, the Mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain exactly two defective parts? **Ans: 285 approximately [GTU Summer-2015]**

14. A multiple- choice test consists of 8 questions with 3 answers to each question (of which only one is correct). A student answer each question by rolling a balanced die and checking the first answer if he gets 1 or 2, the second answer if he gets 3 or 4, and the third answer if he gets 5 or 6. To get a distinction, the student must secure at least 75% correct answers. If there is no negative marking, what is the probability that the student secures a distinction? **Ans: 0.0197 [GTU Summer-2015]**