

HASMUKH GOSWAMI COLLEGE OF ENGINEERING, VAHELAL

APPLIED MATHEMATICS FOR ELECTRICAL ENGINEERING (3130908)

TUTORIAL 4: PROBABILITY

1. Define: Probability, Exhaustive event, Mutually Exclusive event, Favourable event and equally likely event, Pair wise Independence and Mutually Independence.
2. What is the probability that a leap year selected at random will have 53 Fridays? **Ans: 2/7**
3. A bag contains 5 red, 4 blue and m green balls. If the probability of getting two green balls when two balls are selected at random is 1/7, find m. **Ans: m=6**
4. A fair dice is thrown thrice. Find the probability that the sum of the number obtained is 10. **Ans: 1/8**
5. If the letters of the word 'REGULATIONS' are arranged at random, what is the probability that there will be exactly four letters between R and E? **Ans: 6/55**
6. Prove that for any two events A and B in a sample space S,
 - (a) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$
 - (b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 - (c) $P[(A \cap \bar{B}) \cup (\bar{A} \cap B)] = P(A) + P(B) - 2P(A \cap B) = P(A \cup B) - P(A \cap B)$
7. Prove that for any three events A, B and C in a sample space S,
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$
8. A card is drawn from a pack of 52 cards. Find the probability of getting a queen or a spade or a black card. **Ans: 7/13**
9. If A and B are two independent events, then prove that the probability of their simultaneous occurrence is given by $P(A \cap B) = P(A)P(B)$.
10. If A and B are two independent events, then prove that $A \& \bar{B}$; $\bar{A} \& B$; $\bar{A} \& \bar{B}$ are also independent events.
11. If A and B are two events such that $P(A) = \frac{2}{3}$, $P(\bar{A} \cap B) = \frac{1}{6}$ and $P(A \cap B) = \frac{1}{3}$. Find $P(B)$, $P(A \cup B)$, $P(A/B)$, $P(B/A)$, $P(\bar{A} \cup B)$, $P(\bar{A} \cap \bar{B})$. Also, examine the events A and B are (i) equally likely, (ii) Exhaustive, (iii) Mutually exclusive, and (iv) Independent. **Ans: 1/2, 5/6, 2/3, 1/2, 2/3, 1/6**
12. An urn contains four tickets marked with numbers 112, 121, 211 and 222. One ticket is drawn randomly. Let A_i (i = 1, 2, 3) be the events that the i^{th} digit of the ticket drawn is 1. Show that the events A_1, A_2 & A_3 are pair wise independent but not mutually independent.
13. A person A is known to hit a target in 3 out of 4 shots, whereas another person B is known to hit the same target in 2 out of 3 shots. Find the probability of the target being hit at all when they both try. **Ans: 11/12 [Summer 2015]**
14. There are two bags. The first bag contains 5 red and 7 white balls and the second bag contains 3 red and 12 white balls. One ball is taken out at random from the first bag and is put in the second bag. Now, a ball is drawn from the second bag. What is the probability that this last ball is red? **Ans: 41/192**
15. An urn contains 10 red, 5 white and 5 blue balls. Two balls are drawn at random. Find the probability that they are not of the same colour. **Ans: 25/38**
16. State and Prove Baye's Theorem.
17. A company has two plants to manufacture hydraulic machines. Plant I manufactures 70% of the hydraulic machines, and Plant II manufactures 30%. At Plant I, 80% of hydraulic machines are rated standard quality; and at Plant II, 90% of hydraulic machines are rated standard quality. A machine is picked up at random and is to be of standard quality. What is the chance that it has come from Plant I? **Ans: 0.6747 [Summer 2015]**
18. An urn I contents 1 White, 2 Red and 3 Black balls; An urn II contents 2 White, 3 Red and 1 Black balls; An urn III contents 3 White, 1 Red and 2 Black balls. One urn is chosen at random and two balls are drawn. They happen to be white and red. Find the probability that they came from (i) Urn I, (ii) Urn II, and (iii) Urn III. **Ans: 2/11, 6/11, 3/11**