Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2021

Subject Code:3140610 Date:07/09/2021

Subject Name:Complex Variables and Partial Differential Equations

Time:02:30 PM TO 05:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Find the real and imaginary parts of $f(z) = iz + 2\bar{z}$.	**03**
	(b)	State De-Movire's formula and evaluate $(1+i)^{96} + (1-i)^{96}$.	**04**
	(c)	Define analytic function. Show that $u(x,y) = x^2 - y^2 - y$ is harmonic function, find a harmonic conjugate $v(x,y)$.	**07**
Q.2	**(a)**	Find the image of the region $\|z\| < 2$ under the transformation $w = 3z + i$.	**03**
	(b)	Find all solution of $sinz = 2$.	**04**
	(c)	Expand $f(z) = \frac{1}{(z+4)(z+2)}$ valid for the region (i) $\|z\| < 2$ (ii) $2 < \|z\| < 4$ (iii) $\|z\| > 4$.	**07**
		OR	
	(c)	Determine the Mobius transformation which maps $z_1 = -1, z_2 = i, z_3 = 1$ into $w_1 = 0,\ w_2 = i,\ w_3 = \infty$.Hence, find the image of $\|z\| < 1$.	**07**
Q.3	**(a)**	Check whether the function $f(z) = x^2 + y^2 - i2xy$ is analytic or not.	**03**
	(b)	Evaluate $\oint_C \frac{e^z}{z(z-1)}\, dz$ around the circle C: $\|z\| = 2$.	**04**
	(c)	Evaluate the followings: (i) $\int_C \frac{\cos \pi z^2}{(z-2)(z-1)}\, dz$, counter clockwise around the circle C: $\|z\|$=3. (ii) $\int_C \frac{dz}{z^2+4}$, where C is the unit circle.	**07**
		OR	
Q.3	**(a)**	Show that the function $f(z) = \bar{z}$ is nowhere differentiable.	**03**
	(b)	Expand $f(z) = z^2 \exp\left(\frac{1}{z}\right)$ in Laurent's series about $z = 0$ and classify the singularity.	**04**
	(c)	Using residue theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 3\sin\theta}$.	**07**
Q.4	**(a)**	Find the value of $\int_0^{1+i} (\bar{z})^2\, dz$ along the line $y = x$.	**03**
	(b)	Solve the partial differential equation $\left(D^2 - 3DD' + 2D'^2\right)z = \sin(x + 2y)$.	**04**
	(c)	Obtain the complete integral of the followings: (i) $q - p + x - y = 0$. (ii) $q^2 = z^2p^2(1 - p^2)$.	**07**
		OR	
Q.4	**(a)**	Classify the Partial differential equation $\frac{\partial^2 u}{\partial x^2} + 7\frac{\partial^2 u}{\partial x \partial y} + 9\frac{\partial^2 u}{\partial y^2} = 0$.	**03**

	(b)	Solve $(x^2 - yz)p + (y^2 - xz)q = z^2 - xy$.	**04**
	(c)	Find the general solution of the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 25\,\frac{\partial^2 u}{\partial y^2}$ by method of separation of variables.	**07**
Q.5	**(a)**	Solve $px + qy = 3z$.	**03**
	(b)	Using Charpit's method, solve $q = 3p^2$.	**04**
	(c)	A tightly stretched string with fixed end points at $x = 0$ and $x = 20$ is initially given by the deflection $f(x) = kx(20 - x)$. If it is released from this position, then find the deflection of the string.	**07**

OR

Q.5	**(a)**	Solve $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$.	**03**
	(b)	Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = y$.	**04**
	(c)	A rod of 30 cm long has its ends A and B are kept at 20°C and 80°C respectively until steady state conditions prevail. The temperature at each end is suddenly reduced to 0°C and kept so. Find the resulting temperature $u(x, t)$ from the end A.	**07**

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